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ABSTRACT

Does conceptually oriented instruction jeopardize students' computational competence? If it does, then why are so many reform efforts continuing to emphasize the importance of teaching for conceptual understanding? If it does not, then why are the majority of teachers at all grade levels continuing to teach for computational competence without conceptual understanding? This paper presents the results of a computational test taken by two groups of students in ninth-grade general mathematics classes. One group of students practiced computational procedures without an emphasis on the mathematical concepts. The second group of students learned the mathematical concepts underlying the procedures and spent little, if any, time on practicing computational procedures. The findings of the computational test showed that in one conceptually oriented class the average grade-level equivalence for computational competence was increased from a 6.5 grade level at the start of the school year to a 9.1 grade level at the end of the year. One computationally oriented class had an average grade level at the start of the school year of 7.1 and at the end of the school year the grade-level equivalence was 7.5. This was a gain of less than half a year, even though the students spent the whole year practicing computational procedures. Other findings showed that the students in the conceptually oriented classes attempted more items on the posttest than did students in the computationally oriented classes. Furthermore, statements made by the students at the end of the year indicated that they felt they had learned more mathematics in the conceptually oriented class than they had in any of their previous mathematics classes. The results of this study suggested that conceptual understanding enhanced students' computational competence and promoted more positive attitudes towards mathematics. The results also suggested that computational procedures are neither learned nor retained through drill-and-practice exercises without conceptual understanding. (Comparative test results are appended. The text contains 15 tables.) (Author)

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Anne L. Madsen and Perry E. Lanier

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Abstract

Does conceptually oriented instruction jeopardize students' computational competence? If it does, then why are so many reform efforts continuing to emphasize the importance of teaching for conceptual understanding? If it does not, then why are the majority of teachers at all grade levels continuing to teach for computational competence without conceptual understanding? This paper presents the results of a computational test taken by two groups of students in ninth-grade general mathematics classes. One group of students practiced computational procedures without an emphasis on the mathematical concepts. The second group of students learned the mathematical concepts underlying the procedures and spent little, if any, time on practicing computational procedures.

The findings of the computational test showed that in one conceptually oriented class the average grade-level equivalence for computational competence was increased from a 6.5 grade level at the start of the school year to a 9.1 grade level at the end of the year. One computationally oriented class had an average grade level at the start of the school year of 7.1 and at the end of the school year the grade-level equivalence was 7.5. This was a gain of less than half a year, even though the students spent the whole year practicing computational procedures.

Other findings showed that the students in the conceptually oriented classes attempted more items on the posttest than did students in the computationally oriented classes. Furthermore, statements made by the students at the end of the year indicated that they felt they had learned more mathematics in the conceptually oriented class than they had in any of their previous mathematics classes. The results of this study suggested that conceptual understanding enhanced students' computational competence and promoted more positive attitudes towards mathematics. The results also suggested that computational procedures are neither learned nor retained through drill-and-practice exercises without conceptual understanding.

THE EFFECT OF CONCEPTUALLY ORIENTED INSTRUCTION ON STUDENTS' COMPUTATIONAL COMPETENCIES

Anne L. Madsen and Perry E. Lanier[1]

Are students' computational competencies jeopardized when teachers teach for conceptual understanding and reduce the amount of time spent on drill-and-practice exercises? The National Research Council (NRC, 1989) in *Everybody Counts* reported that the prevailing myth is that mathematics learning is viewed as the mastery of basic arithmetic skills:

> Myth: Learning mathematics means mastering an immutable set of basic skills.
> Reality: Practice with skills is just one of many strategies used by good teachers to help students achieve the broader goals of learning. (p. 57)
>
> Myth: The way to improve students mathematical performance is to stress the basics.
> Reality: Basics from the past, especially manual arithmetic, are of less value today than yesterday--except to score well on tests of basic skills. Today's students need to learn *when* to use mathematics as much as they need to learn *how* to use it. Basic skills for the twenty-first century include more than just manual mathematics. (p. 63)

As noted by Colburn (1989), these myths are so strongly held by teachers, students, school administrators, and parents that they have contributed to the lack of any significant improvements in the mathematical curriculum and instructional practices. Efforts need to be undertaken to change these myths to the realities of learning and teaching mathematics.

> It is widely recognized that written computation dominates the instructional program in elementary school mathematics. Paper-and-pencil computation is also prominent in the public's perception of what it means to be mathematically proficient. . . . The public will be skeptical about any proposals for de-emphasizing computation. Those having vested interests in . . . the traditional curriculum will also be resistant to such change. (p. 43)

Changing the perception of mathematical proficiency suggests the following question to be addressed: What evidence is there that conceptual understanding of arithmetic procedures enhances students' computational performance? This question is addressed in this paper by comparing the performance of two groups of students on a computational test of arithmetic operations. One group of students learned the mathematical concepts of the computational procedures, while the other group of students practiced computational procedures throughout the year with no understanding of mathematical

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[1]Anne L. Madsen, assistant professor in the Department of Curriculum and Instruction at the University of Texas at Austin, was formerly a research assistant with the General Mathematics Project. Perry E. Lanier, professor of teacher education at Michigan State University, was coordinator of the project. The data used in this study was collected as part of the project.

concepts. In addition to comparison of the test results of the two groups, students' comments about what they learned in the conceptually oriented class are presented along with the instructional strategies used by the teacher of the conceptually oriented classes.

The Teachers and The Students

> Current research in the United States indicates that classroom instruction is often dominated by teacher lectures, traditional workbook and textbook materials that is mostly drill-and-practice and that little time is left for students to participate actively in the learning enterprise. (Educational Testing Service, p. 51)

Pamela Kaye and Don Green (pseudonyms) taught in a semi-rural/suburban high school with an enrollment of approximately 800 students. Both were excellent teachers, each having over 15 years of classroom experience. Each taught two ninth-grade general mathematics classes. In previous years the general mathematics curriculum for Pamela and Don consisted of drill-and-practice reviews of the basic arithmetic operations with whole numbers, fractions, decimals, and some limited work in percents (see Table 1). However, Pamela volunteered to participate in a four-year project in which she worked with mathematics educators and other teachers to improve the curriculum and instruction in general mathematics classes (see Table 2). The changes she implemented focused on teaching mathematics for conceptual understanding. Pamela Kaye's students learned concepts and the meanings of the operations through problem solving, activity-based tasks, and cooperative learning assignments. She did not include drill-and-practice exercises in her curriculum. Don, in contrast, did not participate in the project and continued to teach his general mathematics classes in a procedurally oriented mode. His general mathematics curriculum consisted of arithmetic reviews, and every day his students practiced computational procedures from the textbook or a mimeographed worksheet.

Although their classes were held during different periods of the day, the number of students in the classes was about the same--from 24 to 30. The achievement levels of the students in each class ranged from a grade equivalent level of 3.1 to HS (high school) on the Iowa Test of Basic Skills, administered at the end of the eighth grade. The students in the general mathematics classes ranged in age from 14 to 17 years, with the majority of students being 15 years old. The only difference in the students in the four classes was that in Pamela Kaye's fifth-period class there were several mainstreamed special education students.

Table 1

The General Mathematics Curricula in Don Green's and Pamela Kaye's Classes Prior to Her Curricula Changes

Whole Number Reviews

Add, Subtract, Multiply, Divide

Decimal Reviews

Add, Subtract, Multiply, Divide

Fraction Reviews

Add, Subtract, Multiply, Divide

Review of Percents

Computations

Geometry (if time permits)

Measurement, Shapes,
Area Formulas

Source: Madsen-Nason, 1989, p. 141

Table 2

Pamela Kaye's Conceptually Oriented General Mathematics Curriculum

FIRST SEMESTER

Introduction to Concepts
- Surface Area and Volume
- Making Manipulatives
- Tangrams

Problem Solving Strategies
- Guess and Check
- Making Tables
- Finding Patterns

Factors and Multiples

Fraction Bars-Introduction
- Manipulatives
- Fraction Concepts

Fraction Concepts
- Part-Whole Relationship

Decimal Concepts
- Part-Whole Relationship

Review & Examinations

SECOND SEMESTER

Percent Concepts
- Part-Whole Relationship

Probability

Similarity

Algebra
- Integers
- Operations
- Symbols
- Formulas

Review & Examinations

Source: Madsen-Nason, 1989, p. 145

During the first year that Pamela Kaye participated in the improvement project, she wondered whether a conceptually oriented approach was in any way detrimental to her students' computational competencies. To answer this question, she gave her students a computational test at the start of school, the end of the first semester, and the end of the school year. In the second year she asked Don Green to give the test to his students as well. He agreed. Pamela Kaye continued to use the computational test to assess her students' progress and growth in arithmetic computations in the following years. Don Green did not teach any general mathematics classes after the year he and Pamela gave the computational test. The results of Pamela's and Don's two general mathematics classes are compared for the year in which Don participated.

Every year one of Pamela Kaye's general mathematics classes was observed on a regular basis. The test results of each of the observed general mathematics classes are reported. The purpose of reporting the results over four years was to find out whether the curricular changes Pamela made in the first and second years were sustained over time. These results are compared with those of Don's third-period general mathematics class in the second year. Don's third period was selected because it was similar (in size and student characteristics) to Pamela Kaye's observed classes. The purpose for comparing the computational achievement of students in a computationally oriented class and in a conceptually oriented class was to ascertain whether conceptually oriented instruction enhanced or jeopardized students' computational competencies.

The Computational Test Results

Pamela Kaye used the *Shaw-Hiehle Computational Skills Test: Grades 7-9* (1972) to measure students' computational competencies. The test consisted of 60 computational problems in two forms (Form A and B). The two forms differed only in the numbers that were used for the problems. There was no difference in their level of difficulty. In each class half the students were given Form A and half Form B. The students were required to apply arithmetic procedures to solve problems involving whole numbers, fractions, and decimals. There were 10 additional problems on percents and another 10 on practical arithmetic problems. The test lasted about 45 minutes.

Total Test Results

The Shaw-Hiehle Computational Skills Test contained 60 items. Table 3 shows the results of the mean number of correct responses and mean number of items attempted on the total test for Pamela Kaye's and Don Green's classes. The mean number of correct items on the pretest was lower for Pamela Kaye's classes than for Don's classes. However, the posttest scores were higher in Pamela Kaye's classes. Her students raised their posttest mean by 15.6 and 13.4. The posttest means in Don Green's classes were raised by only 1.8 and 3.6 respectively.

The mean pretest scores for Pamela Kaye's second- and fifth-period classes were 28.2 (*SD* 7.1) and 22.2 (*SD* 8.2), respectively. The mean pretest scores for Don Green's third- and fourth-period classes were 31.8 (*SD* 7.7) and 31.2 (*SD* 10.0), respectively. The posttest means in Pamela's classes were 43.8 (*SD* 8.8) and 35.4 (*SD* 9.9), and the posttest means in Don Green's classes were 33.6 (*SD* 10.4) and 34.8 (*SD* 9.5). Appendix A shows the frequencies of scores for the pretest, interim, and posttests in all four classes. The results of Don Green's classes showed gains from the pretests to interim tests and losses from the interim tests to the posttests. Pamela Kaye's classes showed gains from the pretests to the interim tests and from interim tests to the posttests as well.

There were three ways to answer the problems on the Shaw-Hiehle Test: (1) compute the answer correctly; (2) attempt an answer, but get it wrong; and (3) not attempt to answer the problem at all. The mean number of items attempted was the sum of the problems tried by the students--it did not matter whether the problems were correct or not, as long as they attempted an answer. The mean number of items attempted on the pretest was nearly the same for Pamela's and Don's classes. A characteristic typical of most general mathematics students is that if they think a problem is too difficult, or they can't remember the procedure, they won't even attempt to answer it. However, Pamela's classes had raised the mean number of items attempted by 8.4 and 4.2 respectively. In Don's third-period class the mean number of items attempted on the posttest was less than the pretest. In his other class the mean number of items tried remained the same.

Figure 1 shows the pretest, interim test, and posttest class means for the Total Test in each of Pamela's classes that were observed regularly and for Don's third-period general mathematics class. The

Table 3

Shaw-Hiehle Computational Skills Test: Total Test Class Means (60 Items)

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	28.2	43.8	50.4	58.8
SD	7.1	8.8		
5th Period	22.2	35.4	49.2	53.4
SD	8.2	9.9		
Don Green				
3rd Period	31.8	33.6	49.2	48.0
SD	7.7	10.4		
4th Period	31.2	34.8	49.8	49.8
SD	10.0	9.5		

results showed the computational achievement of the students in all of Pamela's classes continued to progress during each year. In Don's class, the computational achievement improved during the first semester then dropped during the second semester.

Pamela Kaye's students performed better than did Don Green's on the posttest and her students showed more pretest-to-posttest gain in their computational achievement than did Don's. Table 4 shows the Shaw-Hiehle Computational Test Grade Equivalencies for the pretest and posttest scores for Pamela's second-period class and Don's third-period class during the second year (see Appendix B for conversion of raw scores). The average grade equivalence in Pamela's class increased from a grade level of 6.5 to 9.1. This result was a gain of over two and a half years in computational ability during one year. Don's class increased their average grade level equivalence by less than half a year and it remained at the seventh-grade level.

Observations of Pamela's classes indicated the students' attitudes towards mathematics had changed during each year. The students became more confident in their ability to be successful in mathematics and more willing to try new approaches to learning mathematics by the end of the year. They explored mathematical ideas using manipulatives, drew pictures, and wrote about their conjectures in mathematics. Pamela talked about the achievement and attitude changes in her students in the following interview segment:

> Most of my students start out in the beginning of the school year with about a sixth-grade ability level on the Shaw-Hiehle Computation Test. By the end of the year they were at the ninth-grade level. That's still not where I want them to be, but they have gained a lot. There are extreme cases where I have had one student that gained five grade levels in one year. He didn't have any concept of what was going on, but as soon as I showed him a few basic things he went crazy!
>
> The attitude of most of my students when they first come into my class is "People have been telling me this stuff for nine years, and you aren't going to make a difference for me."
>
> I give the students the Shaw-Hiehle Computation Test in the fall and again in January. I don't tell them their scores or their exact grade levels, but I do tell them how much they have improved. I will say, "You have improved a grade level and a half in one semester. Normally I would have expected you to improve by only a half of a grade level in one semester." Suddenly I see a difference in their opinion of what they can do in mathematics. Occasionally this change has been dramatic. (Madsen-Nason, 1989, p. 269)

Whole-Number Competency

> Most IAEP [International Assessment of Educational Progress] countries still emphasize basic whole number operations at age 13. (Educational Testing Service [ETS], p. 46)

Results of the IAEP (ETS, 1992) indicate that even at the eighth-grade level, basic facts with whole numbers were still emphasized in most countries. Computational competency for many people means the successful application of whole number procedures. The Whole Numbers Subtest (see Table 5) measured students' competencies with whole-number calculations.

Table 6 presents the mean numbers of correct items and items attempted for Pamela's and Don's classes in the second year of the study. At the beginning of the year the mean number of correct items of three classes were close (15.6-15.8); the fourth class (with the special education students) had the lowest mean of 13.4. It seemed likely that students would get most of these problems correct, since they had computed with whole numbers for at least seven years. Pamela's classes had increased the mean number of correct items by 0.8 and 2.8 by the end of the school year. One of Don's classes had increased its mean by 0.2, and the other class did not increase the mean. Pamela Kaye's classes increased their means and both had means that were over a mastery level.

The last two columns in Table 6 show the pretest and posttest mean number of items attempted. It is not surprising to find that most items were attempted by the students, since they were more confident working these problems than they were with fraction, decimal, and percent problems. Pamela's classes attempted more problems on the posttest than the pretest. In contrast, Don's third-period class tried fewer problems and his fourth-period class tried the same number of problems as on the pretest.

Figure 2 shows the pretest, interim (January), and posttest means in Pamela Kaye's observed classes over each of four years and Don Green's third-period class. Each of Pamela's classes showed continual progress in whole-number achievement across the year from pretest to posttest. Don Green's class, in contrast, gained more in the first semester and fell in the second. Overall, Pamela's classes showed more gain than did Don Green's in whole-number computation. The continual gains Pamela's classes made over the year reflected the kinds of experiences with whole numbers in which they engaged. These experiences are discussed in the following section.

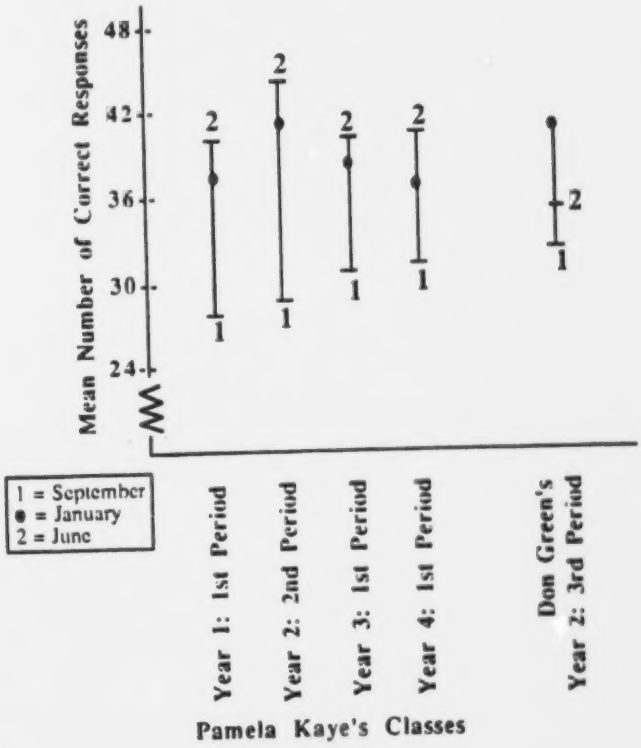

Figure 1. Pre-, interim, and posttest class means on the Shaw-Hiehle Computational Test (60 Items).

Table 4

The Grade Equivalence for the Pretest and Posttest Class Averages in Pamela Kaye's and Don Green's General Mathematics Classes

Classes	Pretest		Posttest	
	Average Raw Score	Grade Equiv-alence	Average Raw Score	Grade Equiv-alence
Pamela Kaye				
2nd Period	28	6.5	44	9.1
Don Green				
3rd Period	32	7.1	34	7.5

Table 5

The Whole Numbers Subtest of the Shaw-Hiehle Computational Skills Test: Grades 7-9 (Form A)

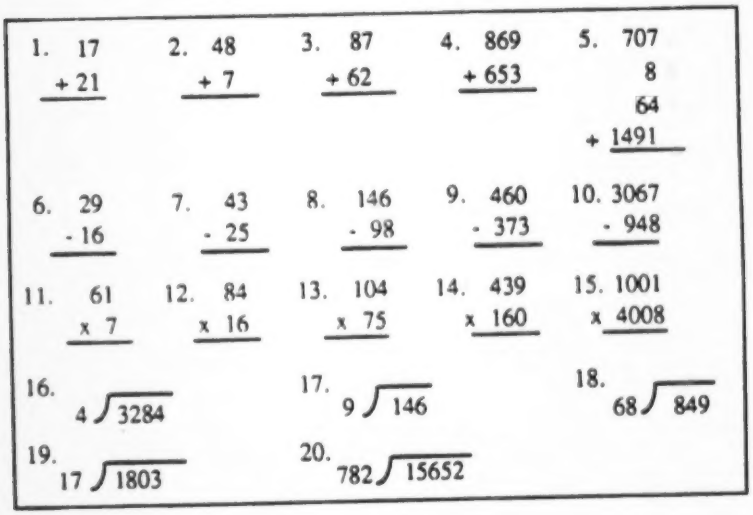

Table 6

The Class Means for the Whole Numbers Subtest (20 Items) of the Shaw-Hiehle Computational Skills Test

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	15.6	16.4	19.6	20.0
5th Period	13.4	16.2	18.4	19.2
Don Green				
3rd Period	15.8	16.0	19.2	18.6
4th Period	15.8	15.8	19.6	19.6

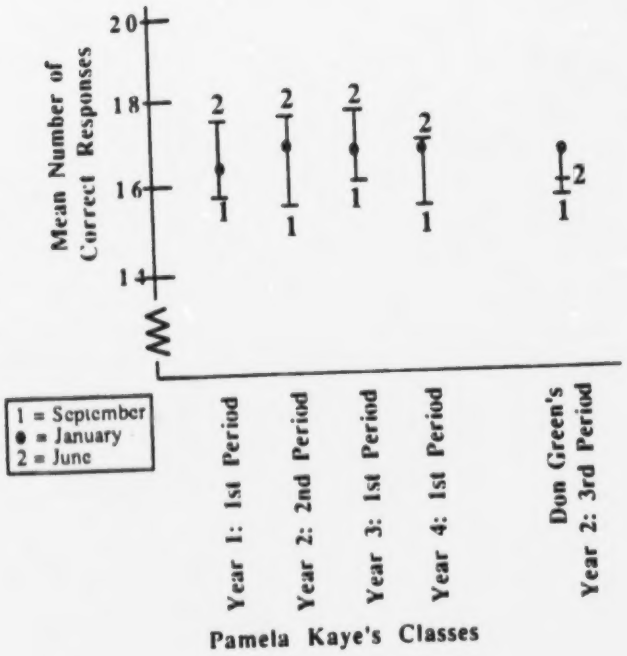

Figure 2. Pretest, interim-, and posttest class means on the whole numbers subtest of the Shaw-Hiehle Computational Skills Test (10 Items).

Pamela Kaye's whole-number instruction. What kind of instructional changes did Pamela implement in her classes that enabled her students to become more successful in whole-number computations? One change she made was to replace whole-number reviews with a unit on problem-solving strategies. She selected the book *Teaching Problem-Solving Strategies* (Dolan & Williamson, 1983) to teach students six problem-solving strategies. Pamela also included activities in estimation over the year. Pamela taught students to use calculators and implemented a calculator activities unit in the first semester. Calculators were used for problem solving and other lessons over the year. Pamela taught the unit *Factors and Multiples* (Lappan, Fitzgerald, Winter, Phillips, & Schroyer, 1986), in which whole-number concepts and relationships were actively investigated. These activities helped students review whole-number computations and developed the concepts needed for understanding fraction operations.

At the end of the school year, Pamela Kaye asked students to tell her what they had learned during the year. Two students replied: "I didn't know the GCF or the LCM before." "I learned stuff like breaking down numbers and the GCF and the LCM and what that meant" (Madsen, 1988).

Students had opportunities to think about the whole-number concepts and operations in many different contexts through estimation, problem solving, calculators, and exploration activities with factors and multiples. Van de Wall (1990) notes,

> For children to be actively engaged in developing alternative, efficient methods of computation requires an understanding of numbers, numeration systems, and meanings of operations. The explorations that build on these concepts will serve to strengthen them. (p. 47)

Fraction Competency

> More time needs to be spent carefully developing the initial fraction concepts and the connections between the concepts and the algorithmic procedures. . . . The purpose for readjusting formal instruction in computation is to allow for a stronger development of fraction concepts and the meanings of the operations. (Coburn, 1989, p. 51)

The ninth-grade general mathematics students had worked on computational problems with fractions since the fourth grade. Yet, remembering the correct computational procedures was always problematic for them. The common fractions subtest, shown in Table 7, consisted of the 10 typical computational problems.

The class means for the pretest and posttest are in columns 1 and 2 of Table 8. The mean number of problems attempted by the students on the pretest and posttest are in columns 3 and 4. On

the pretest Pamela's fifth-period class had the lowest mean and her second-period class mean was between the means of Don's classes. The posttest means for Pamela's classes were both higher than those of Don's classes. The means in Pamela's classes had increased by 3.7 more problems correct on the posttest. The means in Don's classes had increased 2.3 and 1.9 more problems correct. The pretest to posttest gains in Pamela's classes were nearly double the gains made in Don's classes. Pamela Kaye's students attempted more items on the posttest than they did on the pretest. A comparison of the pretest and posttest mean number of problems attempted reveals that the students in her fifth-period class (with the mainstreamed students) attempted more problems from pretest to posttest than did students in any of the other classes. The students in Don's classes attempted fewer problems on the posttest than they did on the pretest.

The pretest, interim, and posttest mean number of correct problems for Pamela Kaye's general mathematics classes that were observed for four years and Don Green's third-period class in the second year are compared in Figure 3. Pamela's instruction in fractions occurred during October and November each year. Don's instruction in fractions also took place also during the first semester. In all four years Pamela's classes continued to gain in fraction achievement, whereas Don's students reached their highest achievement gain at the end of the first semester. The mean number of correct problems on the posttest in Don's class fell below the interim test average. In every case, the posttest means for Pamela's classes was higher than Don's posttest mean.

Pamela Kaye's fraction instruction. How did Pamela Kaye teach fractions that contributed to her students' computational gains? Several instructional strategies were implemented which provided students with opportunities to understand fractions in many ways. She required students to use manipulatives and illustrations to show fraction concepts and operations. In the first year, her students made and used fraction circles to study fraction operations (Madsen-Nason & Lanier 1986). Pamela found other fraction materials in the school district's resource center which were used in following years. These materials included the teacher guide and student activity book from *Fraction Bars* (Bennett & Davidson, 1981).

Table 7

The Problems on the Common Fractions Subtest of the Shaw-Hiehle Computational Skills Test: Grades 7-9 (Form A)

21. $\frac{1}{4} + \frac{3}{8} =$	22. $\frac{2}{3} + \frac{5}{6} + \frac{5}{12} =$	23. $3\frac{2}{3} + 2\frac{3}{5}$
24. $\frac{4}{5} \cdot \frac{1}{5} =$	25. $\frac{3}{4} \cdot \frac{1}{5} =$	26. $3\frac{2}{3} - 1\frac{3}{4}$
27. $\frac{1}{5} \times \frac{2}{7} =$	28. $4\frac{2}{5} \times 15 =$	
29. $5 + \frac{1}{4} =$	30. $2\frac{2}{5} + 6 =$	

Table 8

The Class Means for the Common Fractions Subtest (10 Items) of the Shaw-Hiehle Computational Skills Test

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	2.7	6.4	9.6	9.8
5th Period	1.5	5.2	9.1	9.7
Don Green				
3rd Period	2.2	4.5	8.5	8.1
4th Period	3.2	5.1	9.3	8.8

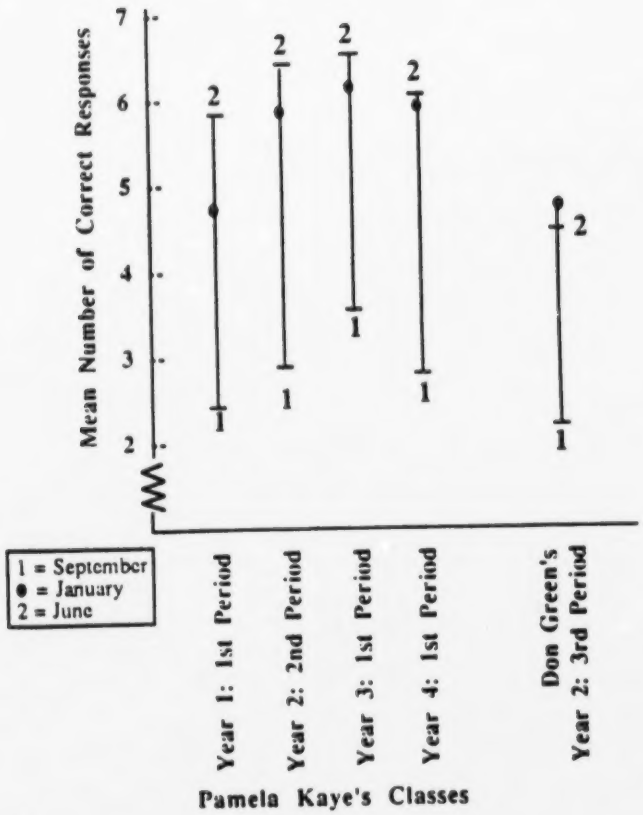

Figure 3. Pre-, interim, and posttest class means on the common fractions subtest of the Shaw-Hiehle Computational Skills Test (10 Items).

Her students made their own set of fraction bars and used them throughout the fraction unit. Pamela continually emphasized the importance of having students talk about fraction concepts and operations. The overarching concept guiding Pamela's instruction was the interpretation of a fraction as a part of a whole. This idea was also emphasized in the decimal and percent units. The students used region, set, and linear models to represent fraction ideas. Pamela added estimation and problem-solving activities in this unit which enabled students to work with fractions in contexts other than simple calculations.

At the end of the second year the students were asked to describe what they learned during the year that was new to them and what mathematics they now understood better. Many students reported that they learned a lot more about fractions. The following are some of their comments (typed as written):

> I learned how to + and x fractions. I understand everything a hole lot better.
>
> ya made fractions better to understand
>
> this year I learned how to add subtract, multiply and divide fractions. I never really understood them before.
>
> Last year I didn't understand fractions, but now I understand them perfictly.
>
> I learned how to do the adding and subtracting of fractions better and I didn't know how last year. Decimals are a breeze now.
>
> I learned a lot about fractions. I think I still need some help. I learned more about decimals. Mainly, I understand. (Madsen, 1988)

Pamela Kaye was asked how important she felt it was for students to develop skills in fraction computation. Her reply focused on the importance of the fraction concepts that the students learned:

> I think fraction concepts are very important, I use them in the Probability and Similarity Units. I still think computation is important, but not as important as before. The concepts are real important because they will take you into decimals, percents and so forth. There is no way they can do percents adequately until they have a grasp of both fractions and decimals. (Madsen-Nason, 1989, p. 244)

Decimal Competency

> The meaningful development of decimal computation is just as important as computation with fractions and whole numbers. (Coburn, 1989, p. 51)

The Decimal Fractions subtest consisted of the 10 typical decimal calculation problems shown in Table 9. The mean number of correct responses and number of problems attempted by the students in Pamela Kaye's and Don Green's classes are shown in Table 10. Pamela Kaye's classes had the lowest

mean number of correct responses on the pretest. However, both classes raised their means by 2.3 and 1.7 on the posttest. The posttest means of Don Green's classes had increased by only 1.0 and 0.9. The mean number of items attempted on the posttest in Pamela's classes were higher than on the pretest. This was not the case in Don's classes, where the means on the posttest were lower than on the pretest.

When the means of Pamela Kaye's observed classes were compared across the years (see Figure 4), it is noted that, even though her students had worked on decimals in the first semester, they continued to make gains in decimal computation in the second semester. In Don's classes, the mean gains made in the first semester had fallen in the second semester.

Pamela Kaye's decimal instruction. Why did Pamela's students continue to improve their competency in decimal computation in the second semester? Part of the answer was that Pamela emphasized the conceptual connections between fractions and decimals and had students express decimals in both decimal and fraction form. She continually focused on the part/whole interpretation of decimals and linked this to fractions. In addition, Pamela used activities and materials from the *Decimal Squares* (Bennett, 1981). This was a companion workbook to the *Fraction Bars* (Bennet & Davidson, 1981) materials. The students used 100-square grids as a way to develop an understanding of decimal concepts and operations. Pamela discovered that, when her students used the grids to illustrate decimal operations, they didn't make the mistakes her students had made in the past. Pamela talked about the importance of using the decimal square illustrations to help students understand decimal concepts.

> Pamela said yesterday was the first day the students started working with addition of decimals and she was having them draw pictures of two decimals and put the two pictures together to come up with the sum. She represented nine-tenths as ninety-hundredths of the decimal square and three-tenths as thirty-hundredths shaded on a decimal square. When the students combined the decimal squares, they realized they had one full square and part of another one.

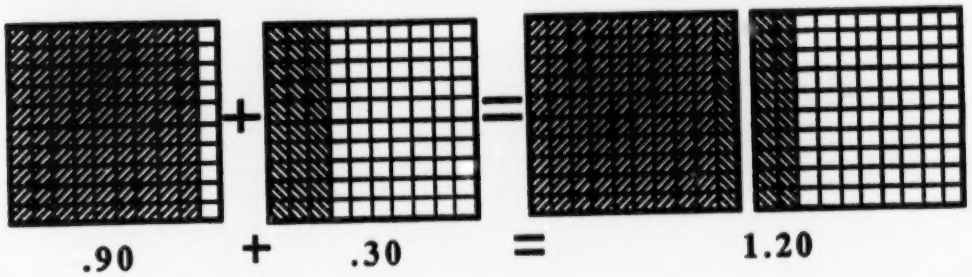

Table 9

The Items on the Decimal Fractions Subtest of the Shaw-Hiehle Computational Skills Test: Grades 7-9 (Form A)

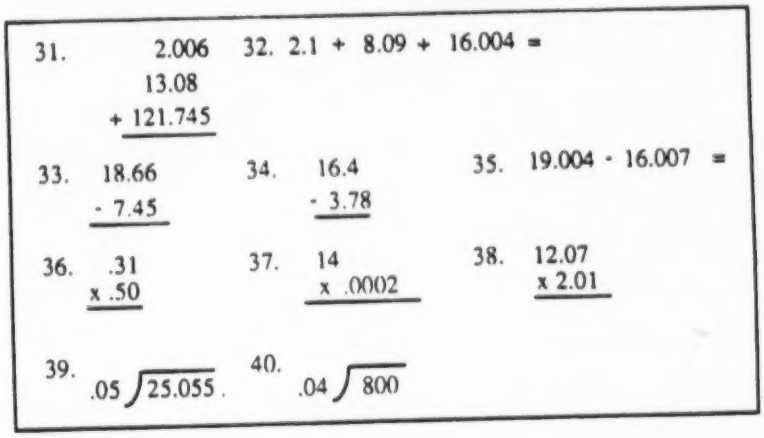

Table 10

The Class Means for the Decimal Fractions Subtest (10 Items) of the Shaw-Hiehle Computational Skills Test

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	5.2	7.5	9.7	10.0
5th Period	4.5	6.2	9.2	9.5
Don Green				
3rd Period	5.8	6.8	9.2	9.0
4th Period	5.6	6.5	9.5	9.4

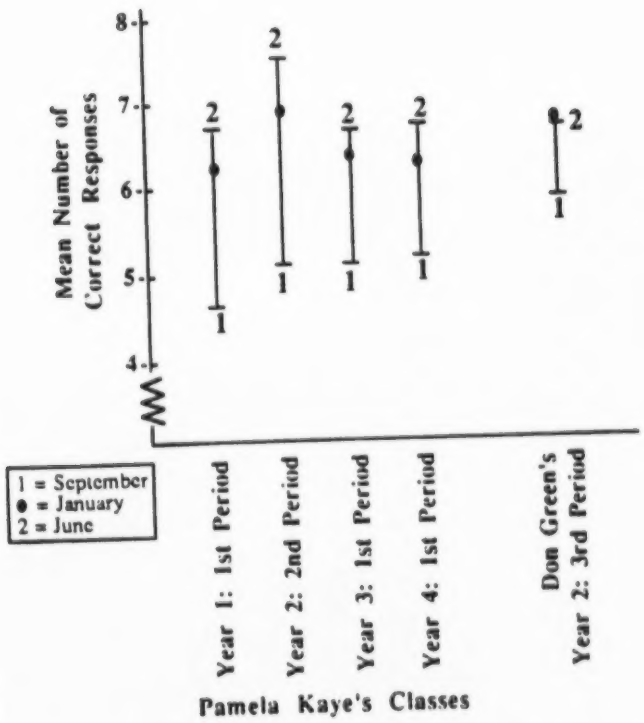

Figure 4. Pre-, interim, and posttest means on decimal fractions subtest of the Shaw-Hiehle Computational Skills Test (10 Items).

> Pamela thought this was a good strategy to get students to see the sum did not equal twelve hundredths (.90 + .30 ≠ .12). She said this had been a common error the students had made in previous years. (Madsen-Nason, 1989, p. 131)

During the second semester her students studied probability and statistics in which decimals and operations with decimals were reviewed. These activities provided opportunities for students to relate decimals to fractions and to percents. Pamela continued her focus on the part/whole interpretations of fractions, decimals, and percents in the second semester. The students talked about having a better understanding of decimals at the end of the school year. The following are student comments (as written) from the second year:

> Decimals were my down fall until you explained them in simple terms. Overall on a scale of 1 to 10 this class gets a 9, because no class is perfact.
>
> Well, I learned how to divide decimals better. Fractions this year were more easier.
>
> I was always lost on decimals in this year they have all been explained to me in a language I can understand.
>
> well I learned alot about fractions and decimals. I knew a little bit but not a lot. I know a lot more about them. I knew a lot about adding and subtracting decimals, but never did any divide or multiplication problems and never did any fractions this way before either.
>
> I didn't understand what the decimals were all about or how to divide and multiply fractions. (Madsen, 1988)

Percent Competency

> One of the main contributors to low levels of achievement with percent is the lack of understanding most students have about the relationships among ratios (expressed as fractions), decimals and percent. . . . Another reason is that we do not take enough "out of book" time to help students develop some fundamental sense of quantity involving parts of a whole. (Coburn, 1989, p. 52)

There were 10 items on the percents subtest (see Table 11). Some items required students to express fractions and decimals as percents and others required them to calculate answers to percent problems. Table 12 describes the mean number of correct items and attempted answers on the percents subtest for Pamela Kaye's and Don Green's classes. The results on this showed the greatest differences between the classes of the two teachers. Pamela Kaye's classes had the lowest pretest means. However, the means in her classes had increased by 4.6 and 2.9 on the posttest. There was no gain in the mean number of correct responses in one of Don's classes and only a gain of 0.9 in the other. The mean number of items attempted was about half for all the classes. Pamela's classes on the posttest attempted most of the items (9.4 and 8.2) while Don's classes still attempted about half the problems.

Figure 5 shows the mean number of correct responses on the pretest, interim test, and posttest. Pamela implemented a conceptual approach in teaching percents during the first semester only. This might explain why her first-year class showed the greatest gain during the first semester. Her students still scored higher on the posttest, even though they did not study percents in the second semester. Pamela taught percents in the second semester in the following years. This accounted for the interim to posttest gains in the class means. The mean in Don's class increased from the pretest to the interim test; however, this gain was lost in the second semester. It seemed as though the students forgot what they learned the first semester.

Pamela Kaye's percent instruction. Pamela avoided teaching percents to the general mathematics students prior to the instructional changes she made. She felt unsuccessful in teaching algorithms for computing answers to percent problems. She believed students remembered the rules only long enough to take the test. Pamela changed the way she taught percents when she changed her curriculum and instruction. She focused on the part/whole interpretation of rational numbers and connected students' experiences with percents to decimals and fractions. She used a 100-percent grid as a model for representing percents. The students shaded in percents on the grids to help them understand the meaning of percent (see Figure 6). When students wrote percents they were required to also write the decimal and fraction equivalent.

Pamela used the article "Another Look at the Teaching of Percents" (Dewar, 1984) to change the way the students calculated percent problems. The article used a 100-percent stick to illustrate percent problems. The students set up a proportion after they illustrated the problems. Once the proportions were set up the students solved the problems. The percent stick gave them a way to estimate the answers to the problems. Figure 7 shows how the 100-percent stick and the proportion were used to solve the problem: 25% of 36 =? Many students used this method to work the percent computations on the posttest.

The probability unit helped Pamela's students connect fraction, decimal, and percent concepts of rational numbers. They frequently expressed probabilities as fractions, decimals, and percents in the unit.

Table 11

The Percents Subtest Items on the Shaw-Hiehle Computational Skills Test: Grades 7-9 (Form A)

41. $\frac{4}{100} = ___\%$	42. $\frac{2}{5} = ___\%$
43. .76= ___ %	44. 50% = ___
45. ___ % of 64 = 16	46. 75% of ___ = 12
47. $66\frac{2}{3}$ % of 27 = ___	48. 150 is ___ % of 100
49. 200 % of 7 = ___	50. 1.2 % of $4000 = ___

Table 12

Shaw-Hiehle Computational Skills Test: Percents (10 Items) Class Means

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	1.7	6.3	4.6	9.4
5th Period	1.4	4.3	5.3	8.2
Don Green				
3rd Period	2.2	2.2	4.9	5.4
4th Period	1.8	2.7	5.0	4.9

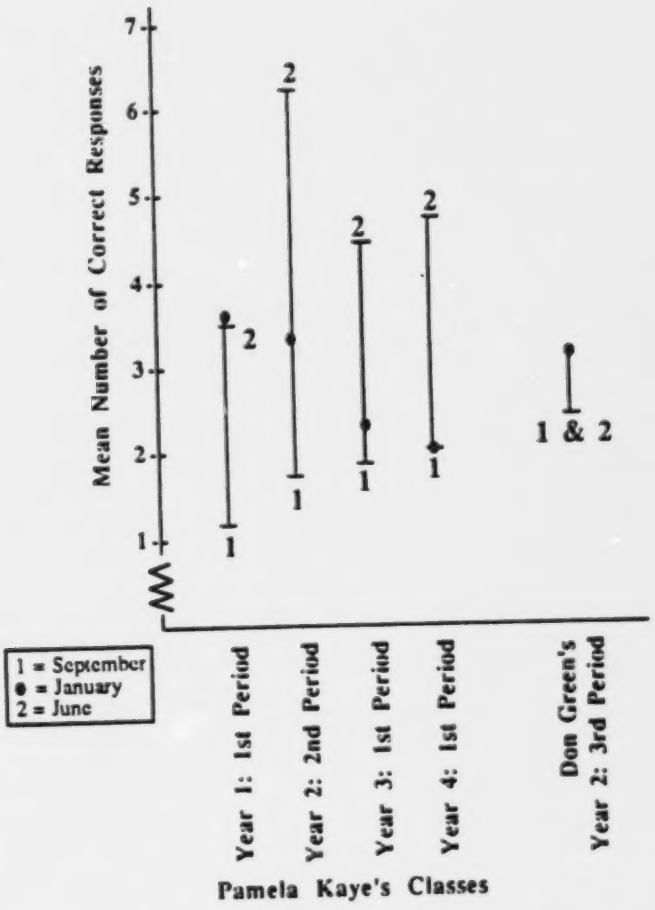

Figure 5. Pre-, interim, and posttest class means on the percent subtest of the Shaw-Hiehle Computational Skills Test (10 Items).

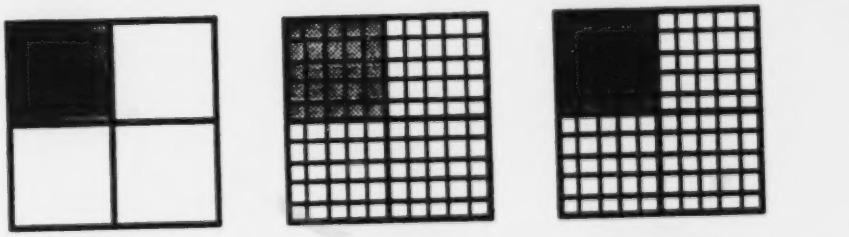

Figure 6. The representations used by Pamela Kaye to link the concepts of fractions, decimals, and percents.

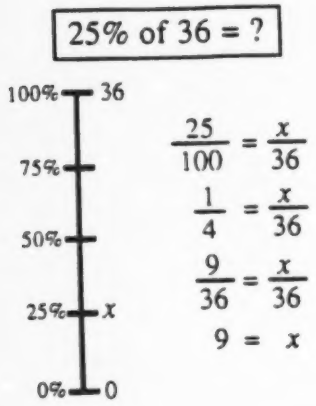

Figure 7. The 100-percent stick and the related proportion used by Pamela Kaye to represent, estimate, and solve percent problems.

The students recognized they had learned a lot about percents and how to solve percent computations. Some of their comments in the second year were as follows:

> Well, I didn't quite understand percents before, but I learned a little. More than I did before.
>
> I didn't understand how to do changing %s to decimals and fractions but now I think I do.
>
> I learned more about decimals than I ever did before. And about percents. (Madsen, 1988)

Computational Competence With Practical Arithmetic Problems

> Modern programs of instruction will concentrate their efforts in the areas of conceptualization and problem solving. (Coburn, 1989, p. 50)

The practical arithmetic subtest included 10 problems where the students applied a formula or procedure to a situation (see Table 13). Table 14 shows the mean number of correct responses and items attempted for Pamela Kaye's and Don Green's classes. The pretest means showed less than half the problems were correct in all classes. Pamela's classes increased the means by 2.7 and 1.1, respectively. Don's third-period class mean had dropped 0.5 and his fourth-period class had increased the pretest mean by only 0.1. Pamela's second-period class had increased the mean number of items attempted by 3.1 from pretest to posttest. Even her fifth period (with the special education students) tried more problems on the posttest than they did on the pretest. Figure 8 shows that the mean number of correct items in Pamela Kaye's observed classes had increased from pretest to posttest. Don's class mean showed a first semester gain, but by the posttest the mean was lower than on the pretest.

Pamela Kaye's practical arithmetic instruction. Pamela omitted the computational reviews of whole numbers, decimals, and percents she used in previous years. She replaced them with problem-solving activities from *Teaching Problem Solving Strategies* (Dolan & Williamson, 1983). The students spent time solving problems in cooperative groups using calculators. They worked on four or five problems focused on one or more problem-solving strategies instead of several pages of computational problems. Pamela used estimation activities to provide students with the opportunity to use estimation to check their answers to problems.

Pamela included problem-solving activities in all her units during the year. Substituting problem-solving activities for drill-and-practice lessons gave students practice in applying computations to practical

arithmetic and problem-solving situations. Students practiced more computations working on problem-solving activities than they did on several drill-and-practice worksheets. Integrating problem solving throughout the year increased students' confidence in their ability to be good problem solvers. At the end of the school year one student's comment related to a problem-solving strategy he learned, "Things I learned this year were--I learned how to do guess and check."

Computational Competence Through Conceptual Understandings

> Emphasizing mathematical concepts and relationships means devoting substantial time to the development of understandings. It also means relating this knowledge to the learning of skills by establishing relationships between the conceptual and procedural aspects of tasks. (NCTM, 1989, p. 17)

Pamela Kaye's classes achieved a level of computational competence because of her conceptually oriented instruction. It is unlikely the students would have achieved the same levels of competency if they were only exposed to drill-and-practice activities. The general mathematics curriculum changed to focus on the development of mathematical concepts to enable students to understand the computational procedures. Earlier, Table 1 described the content of Pamela Kaye's and Don Green's computationally oriented curriculum, and Table 2 illustrated Pamela Kaye's conceptually oriented curriculum.

The students achieved an understanding of the reasons why computations worked and they made connections between the arithmetic operations with whole numbers, decimals, fractions, and percents. Conceptually oriented instruction did not mean that computational competencies were not needed or that procedures were not practiced. Conceptually oriented instruction enhanced students' understandings of computational procedures, increased retention, and raised the level of their computational competency. "Yes, the *Standards* calls for more computation than ever before. But the tedious, drill-oriented, rule-driven, pencil-and-paper emphasis should be substantially decreased" (Van de Walle, 1991, p. 51).

When asked about the role of drill and practice in general mathematics classes, Pamela replied that it was needed but in a different context.

> I think it is real important, but I think it needs to be done differently. If we are talking about the kinds of drill and practice where students sit down by themselves and do drill and practice alone, that is a waste of time. What we need to do is more drill and practice with the whole class, where students are doing more controlled practice during direct

Table 13

The Practical Arithmetic Problems Subtest of the Shaw-Hiehle Computation Skills Test: Grades 7-9 (Form A)

51. What is the cost of 6 pencils at 60¢ a dozen?
52. At 4¢ each, how many pencils can be bought for 24¢?
53. A team won $\frac{1}{4}$ of its games. If they played 20 games, how many did they win?
54. A man drove his car 216 miles on 12 gallons of gas. How many miles did he get to a gallon?
55. Carl spent 25% of his money for some presents. What percent did he have left?
56. If $\frac{1}{4}$ of an inch on a map represents 3 miles, how maile would one inch represent?
57. If a garden is 20 ft. by 35 ft., how many feet of fence are needed to enclose it?
58. How many square feet of carpet are needed to cover the floor of a room 10 ft. by 12 ft.?
59. What do you pay for goods marked $13.50 with a discount of 20%?
60. A man receives a rate of $3.00 per hour for a 40 hour week. If he receives $1\frac{1}{4}$ times the regular rate for overtime, how much will he earn working a 50 hour week?

Table 14

Shaw-Hiehle Computational Skills Test: Practical Arithmetic (10 Items) Class Means

Classes	Mean Number of Correct Responses		Mean Number of Items Attempted	
	Pretest	Posttest	Pretest	Posttest
Pamela Kaye				
2nd Period	3.4	6.1	6.6	9.7
5th Period	2.5	3.6	7.4	7.8
Don Green				
3rd Period	4.6	4.1	7.6	7.1
4th Period	4.0	4.1	6.5	7.1

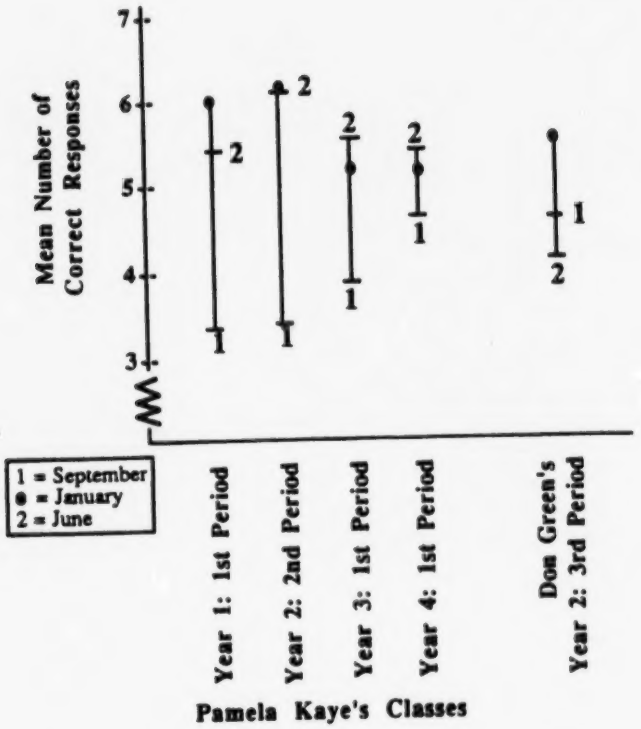

Figure 8. Pre-, interim, and posttest class means on the practical arithmetic subtest of the Shaw-Hiehle Computational Skills Test (10 Items).

> instruction. Drill and practice where we are doing problems together as a group is important. We now work on problems where we try one problem individually and then as a group we look at it and dissect it and decide what was right or wrong about it. I believe that drill and practice can be done within the context of problem solving. (Madsen-Nason, 1989, p. 257)

The students in Pamela's class who were observed in the second year showed that their grade-level equivalency had increased by more than two years during the year (see Table 4). The pretest and posttest total test scores and grade-level equivalencies for three students in Pamela's class are shown in Table 15. Randy, Kenneth, and Karla (pseudonyms) were students with average mathematical ability. The yearly gain in computational competency for each student was more than three years--a result of conceptually oriented instruction (Madsen-Nason & Lanier 1986). Students in Pamela's observed class in the second year commented on what they had learned in her class during the year.

> Just about everything you taught us I have done before but out of a book. A book just tells you how to get the answer but not why.
>
> I learned what GCF [Greatest Common Factor] and LCM [Least Common Factor] were, how to read decimals, how to change decimals into fractions and percents, how to +, -, x, ÷ fractions. I never understood any of this, that is why I hated math. Our teacher would give us page numbers and say good luck. It is more if someone explains it to you. (Madsen, 1988)

Discussion

It is difficult to convince educators to reduce the amount of drill and practice in their mathematics curriculum and include more activities that develop students' understandings of mathematical concepts. Many teachers still believe computational competency is achieved only through drill and practice and that reducing that time would jeopardize students' computational achievement. The results of this study showed that the students in Pamela Kaye's classes achieved computational competency by engaging in experiences designed to develop understanding of mathematical concepts. The computational achievement of students in Don Green's classes was less than that of Pamela Kaye's students even though Don's curriculum was entirely drill-and-practice oriented.

The results of this study suggest that experiences in problem solving, estimation, mental arithmetic, and calculator activities encouraged students to explore arithmetic concepts in many different ways. The inclusion of new mathematical topics such as probability and similarity provided opportunities for students to use computations to solve interesting and challenging problems. Mathematics became

more meaningful and engaging when the students understood the concepts and applied them in problem-solving situations.

During the year, Pamela Kaye's students' success in mathematics changed their attitudes about mathematics. By the end of the second semester they put forth more time and effort to understand mathematical problems and attempted to solve problems they would not have tried at the start of the school year. Pamela Kaye reported that of the 30 students in one general mathematics class, 10 elected to take algebra the following year, and at the end of the year all the students were successful. It is unlikely this outcome would have happened if the students were in a computationally oriented general mathematics class.

When students studied mathematical concepts through a variety of experiences (different mathematical content) in different ways (concrete manipulatives, illustrations, and mathematical symbols), they increased their ability to think about and solve computational problems. The traditional drill-and-practice curriculum provided students with one way to solve a computational problem--apply a memorized algorithm. The conceptually oriented curriculum enabled students to solve computational problems using a number of different strategies.

Table 15

The Pretest and Posttest Grade Equivalence for Three Students in Pamela Kaye's Second Year General Mathematics Class

	Pretest		Posttest	
Student	Raw Score	Grade Level Equivalency	Raw Score	Grade Level Equivalency
Randy	15	4.4	39	8.3
Kenneth	15	4.4	34	7.5
Karla	26	6.6	52	10.2

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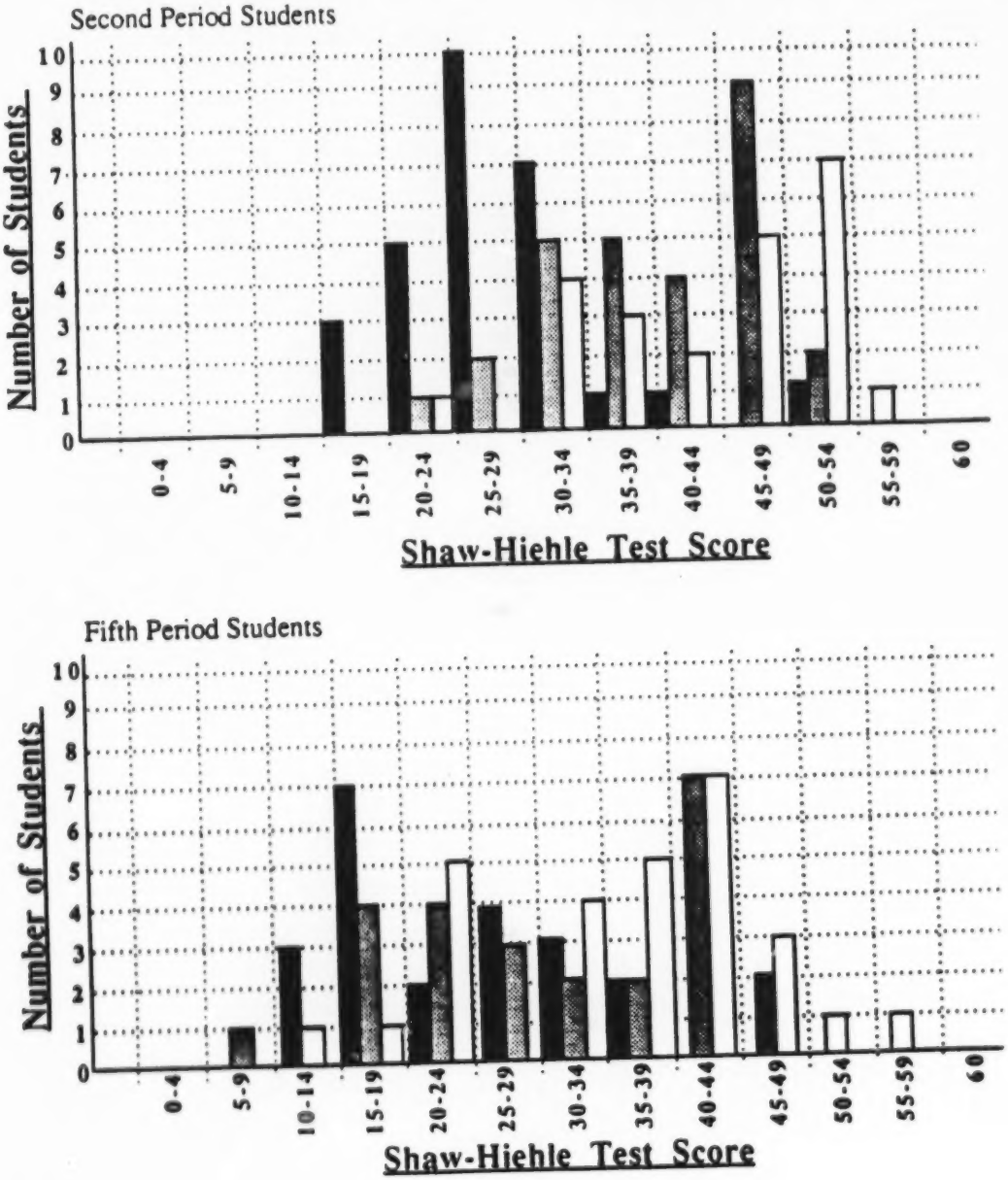
Appendix A

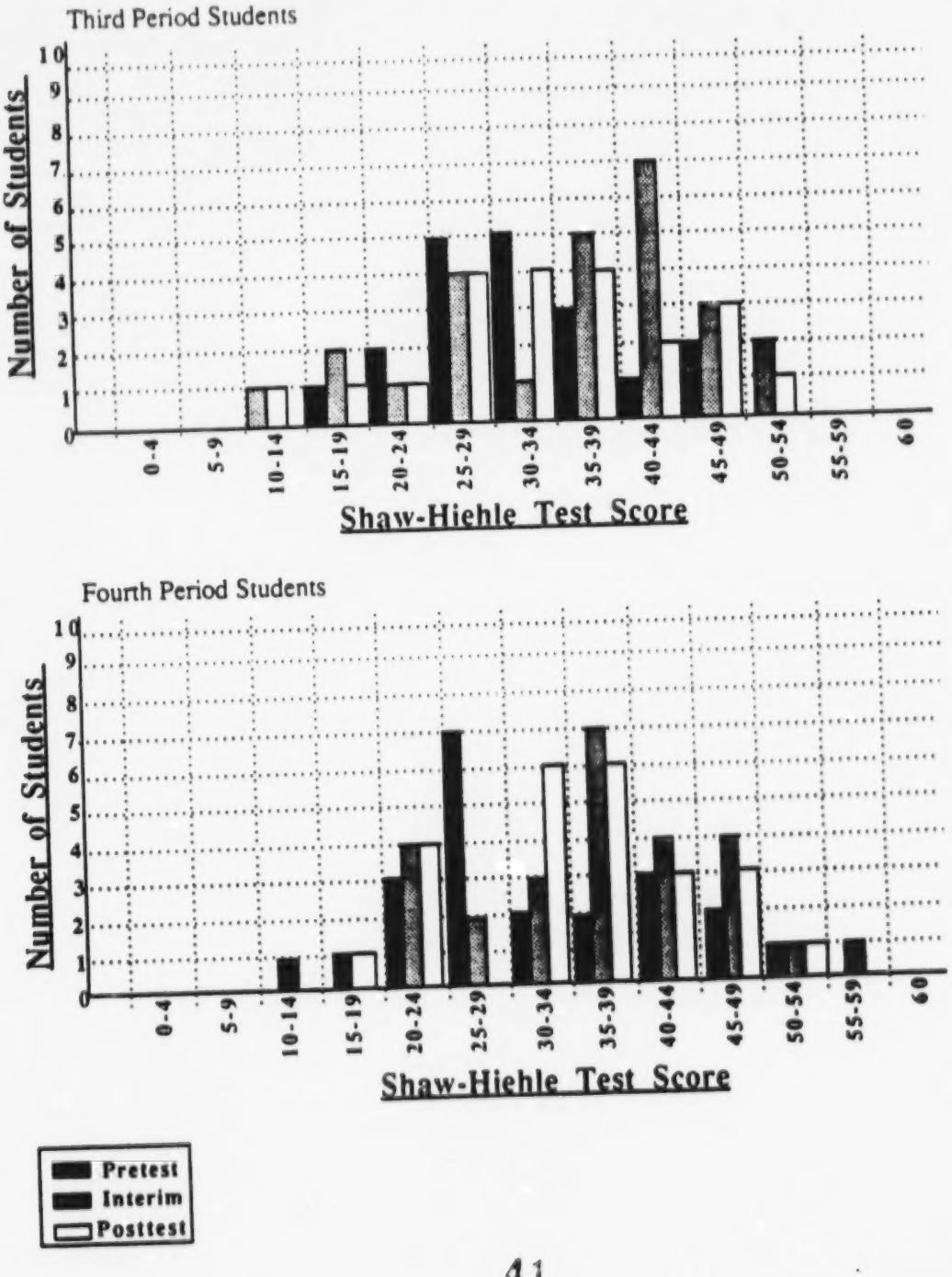
Pretest, Interim, and Posttest Class Results on the Shaw-Hiehle Computational Skills Test

Classes	Pretest		Interim Test		Posttest	
	M	SD	M	SD	M	SD
Pamela Kaye						
2nd Period	28	7.14	40	7.67	44	8.80
5th Period	22	8.16	32	11.62	36	9.90
Don Green						
3rd Period	32	7.65	36	14.40	34	10.41
4th Period	31	10.04	37	14.50	35	9.51

Pretest, Interim, and Posttest Frequencies of the Shaw-Hiehle Test Scores in Pamela Kaye's Second Year Mathematics Classes



Pretest, Interim, and Posttest Frequencies of the Shaw-Hiehle Test Scores in Don Green's Second Year Mathematics Classes



Appendix B

Shaw-Hiehle Computational Skills Test: Grades 7-9 (Forms A & B) Conversion Table - Raw Score to Grade Equivalence

Raw Score	Grade Level Equivalent	Raw Score	Grade Level Equivalent	Raw Score	Grade Level Equivalent	Raw Score	Grade Level Equivalent
1	3.1	16	4.6	31	6.9	46	9.4
2	3.1	17	4.7	32	7.1	47	9.5
3	3.1	18	4.8	33	7.3	48	9.6
4	3.1	19	5.1	34	7.5	49	9.8
5	3.1	20	5.3	35	7.6	50	9.9
6	3.1	21	5.4	36	7.7	51	10.1
7	3.2	22	5.6	37	7.9	52	10.2
8	3.2	23	5.7	38	8.1	53	10.4
9	3.3	24	5.8	39	8.3	54	10.6
10	3.4	25	6.1	40	8.5	55	10.8
11	3.6	26	6.3	41	8.6	56	11.1
12	3.7	27	6.4	42	8.7	57	11.1
13	3.9	28	6.5	43	8.9	58	11.1
14	4.2	29	6.6	44	9.1	59	11.1
15	4.4	30	6.8	45	9.2	60	11.1